University of Florida

GNA QUALIFYING EXAM  JANUARY 8, 2016

Name:

ID #:

Instructor: Maia Martcheva

Directions: This is Part I of the PhD qualifying exam in Numerical Analysis or the semester Exam on MAD6407. If you are taking the PhD qualifying exam, you must take both Part I and Part II (numerical linear algebra) in one sitting. You must show all your work as neatly and clearly as possible and indicate the final answer clearly.

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(1) (20 points) A forth degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24$, $\Delta^3 P(0) = 6$, and $\Delta^2 P(0) = 0$, where $\Delta P(x) = P(x+1) - P(x)$. Compute $\Delta^2 P(10)$. 
(2) (20 points) Consider the fixed point iteration
\[ x_{n+1} = \phi(x_n), \quad n = 0, 1, 2, \ldots \]
where
\[ \phi(x) = Ax + Bx^2 + Cx^3 \]
Given a positive number \( \alpha \), determine the constants \( A, B, C \) such that the iteration converges locally to \( 1/\alpha \) with order \( p = 3 \).
(3) (20 points) This problem has the following parts:

(a) Find $\alpha$, $\beta$ and $\gamma$ so that the quadrature formula has a maximum degree of precision. What is the exact degree of precision of this quadrature formula?

\[
\int_0^2 f(x) \, dx \approx \alpha(f(0) + f(2)) + \beta(f'(0) - f'(2)) + \gamma(f''(0) - f''(2)).
\]

(b) Suggest your own quadrature formula for the integral

\[
\int_0^2 f(x) \, dx.
\]

Your formula should have expected degree of precision at least four (you don’t have to compute to demonstrate that).
(4) (20 points) For the function

\[ f(x) = \sqrt[m]{|x|}, \quad m \neq \frac{1}{2} \]

in the interval \([-1, 1]\), find the polynomial of minimax approximation of degree 2. What is the minimax error? What happens if \( m = \frac{1}{2} \)?
(5) (20 points) Prove that Gaussian quadrature formula (for any $n$) in the interval $[-1, 1]$ and with weight $w(x) = 1$ is exact for all odd functions.