Answer six questions and at least one from each section.

Section 1

1. Sketch the proof of Gödel’s Completeness Theorem for Predicate Logic and explain the use of Henkin constants (or Skolem functions).

2. Show that for any elementary chain \( \{ \mathcal{A}_i \}_{i \in \omega} \) of structures, \( \mathcal{A}_i \) is an elementary submodel of the union \( \mathcal{A} \).

3. Show that the theory of dense linear orderings without end points is complete and decidable.

Section 2

4. Suppose \( 2 \leq \kappa \leq \lambda \) and \( \lambda \) is infinite. Show that \( 2^\lambda = \kappa^\lambda \).

5. State and prove Konig’s Lemma. (Hint: trees.)

6. Show that for any notion of forcing \( \mathcal{P} \) and any countable set \( \mathcal{D} \) of \( \mathcal{P} \)-dense sets, there exists a \( \mathcal{D} \)-generic \( \mathcal{P} \)-filter.

Section 3

7. Sketch a proof of Gödel’s incompleteness Theorem for Arithmetic.

8. Define many-one and truth-table reducibility and show that one implies the other but the converse does not hold in general.

9. State the Friedburg-Muchnik Theorem and sketch the proof. Explain how the “priority” argument deals with “injury”.