Logic exam  

Please complete seven problems among the following, including one from each section.

1. General logic

1A. State the incompleteness theorem and explain the role of Goedel numbering in its proof.
1B. Sketch the proof of the completeness theorem for first order logic.
1C. Sketch the proof of Tarski's undefinability of truth.

2. Model theory

2A. Prove that the theory of dense linear orders without endpoints is complete.
2B. Find two models which are elementarily equivalent but not isomorphic.
2C. State and prove Tarski's criterion for elementary submodels.

3. Set theory

3A. Sketch the proof of $L$ satisfying CH.
3B. Prove from Martin's Axiom that every set of reals of size aleph one has Lebesgue measure zero.
3C. Prove that every stationary subset of omega one can be split into aleph one many pairwise disjoint stationary subsets.

4. Computability

4A. Show how to construct a simple recursively enumerable set $A$ (meaning that $\omega - A$ has no infinite r.e. subset).
4B. Define the recursively enumerable set $K = \{a : \{a\}(a) \downarrow\}$. Use the $S^1_1$ function to show that $K$ is many-one complete.
4C. Prove that the following are equivalent, for any nonempty $A \subset \omega$: (a) $A$ is the domain of some partial recursive function $\phi$. (b) $A$ is the range of some (total) recursive function $f$. 
