I. **General Logic**
1. State and prove the Löwenheim-Skolem Theorem for countable languages.
2. State and prove Herbrand's Theorem.
3. Suppose that $\mathcal{N}$ is a submodel of $\mathcal{M}$. Prove that if $A$ is a quantifier-free sentence, then
   \[ \mathcal{N} \models A \iff \mathcal{M} \models A \]

II. **Set Theory**
4. Prove the Mostowski Collapsing Lemma: If $(M,E)$ is a well-founded model of extensionality, then there is a transitive set $X$ and an isomorphism $\phi : (M,E) \cong (X,\in)$
5. Sketch a proof of $\text{Con}(ZFC) \implies \text{Con}(ZFC + \neg CH + \omega_1 = \omega_1^L)$. You may assume general theorems about forcing, eg. the Truth Lemma.
6. Define the constructible universe $L$ and prove that for all $\alpha$, $L_\alpha$ is transitive.

III. **Recursion Theory**
7. Prove that $A$ is recursive if and only if there is a strictly increasing onto $f : \mathbb{N} \to A$.
8. Prove that there exist recursively enumerable, recursively inseparable sets.
9. Prove that there is no primitive recursive function $F(e,n)$ such that for every primitive recursive function $f(n)$ there is an $e$ such that $F(e,n) = f(n)$ for all $n$. 
