1. Let $D$ be the closure of the unit disk. Assume $f : D \rightarrow \mathbb{C}$ is continuous with $f$ analytic on $D$, $|f(z)| > 3$ for $|z| = 1$, and $f(0) = 1 - 2i$. Must $f$ have a zero in the unit disk? Prove your answer is correct.

2. Let $G$ be a simply connected region. A function $f : G \rightarrow G$ is said to be biholomorphic if $f$ is bijective, analytic, and $f^{-1}$ is also analytic. If $z_1, z_2$ are distinct elements of $G$ and $f_1, f_2 : G \rightarrow G$ are biholomorphisms with $f_1(z_1) = f_2(z_1)$ and $f_1(z_2) = f_2(z_2)$, prove that $f_1 = f_2$.

3. Assume that $f$ is entire, $f(0) = 3 + 4i$, and $|f(z)| \leq 5$ when $|z| < 1$. What is $f''(0)$?

4. Let $\mathcal{F}$ be the collection of analytic mappings of open unit disk $D$ into $\{\text{Re}(z) > 0\}$ such that $f(0) = 1$. Show that $\mathcal{F}$ is a normal family.

5. Let $h_n$ be a sequence of harmonic functions on the connected open set $G$ and assume that $h_n \rightarrow h$ uniformly on compact subsets of $G$. Show that $h$ is harmonic.

6. Assume that $f$ and $g$ are entire and for all $z \in \mathbb{C}$, $|f(z)| \leq |g(z)|$. Show that for some constant $c$ with $|c| \leq 1$, $f = cg$.

7. Assume that $f$ is a meromorphic function on $\mathbb{C}$. A complex number $w$ is called a period of $f$ if $f(z + w) = f(z)$ for all $z \in \mathbb{C}$.

   (a) If $w_1$ and $w_2$ are periods of $f$, show that for all integers $n_1$ and $n_2$, $n_1w_1 + n_2w_2$ is also a period of $f$.

   (b) If $G$ is a bounded region of $\mathbb{C}$, show that $f$ has at most finitely many periods in $G$.

8. Let $p$ be the polynomial $p(z) = a_0 + a_1z + \ldots + a_nz^n$. Show that for each $j = 0, 1, \ldots, n$,

$$|a_j| \leq \max\{|p(z)| : |z| = 1\}.$$  \hspace{1cm} (1)