1. Assume that $h: \mathbb{C} \to \mathbb{R}$ is harmonic, and further that $h(z) > 0$ for all $z \in \mathbb{C}$. Show that $h$ is constant.

2. Prove the Fundamental Theorem of Algebra: if $p(z) = a_0 + a_1 z + \cdots + a_n z^n$ with each coefficient $a_j \in \mathbb{C}$, then there exists a $w \in \mathbb{C}$ with $p(w) = 0$.

3. Recall that $H(G)$ is the set of all holomorphic functions defined on the open, connected set $G$ and that $f_n \to f$ in $H(G)$ exactly when $f_n \to f$ uniformly on all compact subsets of $G$. Assume that the family $\{f_n\} \subset H(G)$ is locally bounded and further that $f \in H(G)$ has the property that the set
$$\{z \in G : \lim_{n \to \infty} f_n(z) = f(z)\}$$
has a limit point in $G$. Show that $f_n \to f$ in $H(G)$.

4. A fixed point of a function $f$ is a point $x$ with $f(x) = x$. Let $D = \{z \in \mathbb{C} : |z| < 1\}$. If $f : D \to D$ is analytic and has two fixed points, show that $f$ is the identity map.

5. If $a_0 \geq a_1 \geq \cdots \geq a_n > 0$, show that $p(z) = a_0 + a_1 z + \cdots + a_n z^n$ has no roots in $|z| < 1$.

6. For which $z$ does the infinite product
$$\prod_{n=0}^{\infty} (1 + z^{2^n})$$
converge?

7. Let $G$ be an open, connected and bounded subset of $\mathbb{C}$. Assume that $f$ is continuous on the closure $\text{Cl}(G)$ of $G$, analytic on $G$, and further that $|f(z)| = 1$ for all $z \in \text{Fr}(G)$ where $\text{Fr}(G)$ is the topological frontier of $G$. Show that either $f$ has a zero in $G$ or else there is a constant $c$ with $|c| = 1$ and $f(z) = c$ for all $z \in G$.

8. Evaluate the integral, and justify each step.
$$\int_{-\infty}^{\infty} \frac{x \sin x}{(x^2 + 1)^2} \, dx.$$