1. Let \( \lambda > 1 \). How many solutions does the equation \( \lambda - z - e^{-z} = 0 \) have in the right half plane \( \{ z : \text{Re} z > 0 \} \).

2. Determine all one-one entire functions.

3. Determine the order of the entire function \( \cosh(\sqrt{z}) \).

4. Does there exist a function analytic in \( |z| < 1 \) satisfying \( |f| < 1 \), \( f(0) = 0 \) and \( f(1/2) = 3/4 \)?

5. Let \( a_n \geq 0 \) and \( f = \sum_{n=0}^{\infty} a_n z^n \) with radius of convergence \( \infty > R > 0 \). Show that \( z = R \) is a singularity of \( f \). (Suggestion: Show, if \( R > r > 0 \) and
\[
\sum \frac{f^{(n)}(r)}{n!} (z - r)^n
\]
has radius of convergence \( R' \), then for each \( |z_0| = r \)
\[
\sum \frac{f^{(n)}(z_0)}{n!} (z - z_0)^n
\]
has radius of convergence at least \( R' \). Conclude \( f \) is analytic in \( |z| < R' + r \).

6. Show that \( f(z) = \sum \frac{z^n}{n^2} \) is analytic at each point \( |z| = 1 \), except \( z = 1 \).

7. Suppose \( f \) is entire of order \( \rho \). Let \( n(r) \) denote the number of zeros of \( f \) in the disc \( |z| < r \). Show
\[
\limsup_{r \to \infty} \frac{\log(n(r))}{\log(r)} \leq \rho.
\]

8. Evaluate the integral
\[
\int_0^{\infty} \frac{t^a}{1 + t + t^2} dt - 1 < a < 1.
\]
(Suggestion: Integrate along the keyhole contour, cut along the nonnegative real axis consisting of the two edges of the real axis from \( \epsilon \) to \( R \) and a small circle of radius \( \epsilon \) and a large circle of radius \( R \) about the origin.)

9. Let \( A \) denote the region inside the circle \( |z - 2| = 2 \) and outside the circle \( |z - 1| = 1 \). Construct a conformal map of \( A \) onto the unit disc. (Suggestion: Where does the map \( 1/z \) carry \( A \)?)