1. Every simple planar graph has a vertex of degree at most 5.

2. (a) State the max-flow min-cut theorem for networks.
   (b) Show that the max-flow min-cut theorem implies the following version of Menger's Theorem.

   The maximum number of edge disjoint directed paths between two given points $s, t$ of a directed graph equals the minimum number of edges whose removal separates $s$ from $t$.

3. We have an unlimited supply of bricks of length 1, each painted red, white or blue, and bricks of length 2, each painted green or yellow. Explicitly find $a_n = \text{the number of ways to choose a sequence of bricks of total length } n$.

4. State and prove Dilworth's Theorem for a finite poset.

5. Each of $n$ gentlemen checks both his hat and his umbrella at a restaurant. Both the hats and the umbrellas are returned randomly. What is the probability that no man gets back both his hat and his umbrella?

6. Prove that

   $$\sum_{i=0}^{m} \binom{s}{i} \binom{t}{k-i} = \binom{s+t}{k}.$$ 

7. 6 identical black balls and $n - 6$ identical white balls are arranged in a row. How many ways are there to do this so that no 3 consecutive balls are black?

8. Define a $q$-Hamming code $C$ as one whose parity-check matrix $H$ has as its columns one non-zero vector from each 1-dimensional subspace of the vector space $GF(q)^r$.
   (a) What is the length of $C$?
   (b) What is the minimum distance in $C$?
   (c) Prove that $C$ is perfect.

9. (a) State the Bruck-Ryser-Chowla theorem.
   (b) Use this theorem to prove that a projective plane of order 6 cannot exist.