Do 6 out of the 7 problems

1. State and prove the Fubini Theorems.
2. State and prove the Vitali integral convergence theorem.
3. Let $\mu$ be a signed measure on the $\sigma$-algebra $\mathcal{S}$. Define positive and negative sets for $\mu$. Prove that if $E \in \mathcal{S}$, then $\mu^+(E) = \sup \{ \mu(F) : F \subseteq E, F \in \mathcal{S} \}$.
4. Let $Y$ be an orthogonal set in a Hilbert space. Prove that $\sum_{y \in Y} y$ converges if and only if $\sum_{y \in Y} \|y\|^2 < \infty$.
5. Let $X$ be a Banach space, $A \subseteq X$. Suppose that $\sup_{a \in A} \|x^*a\| < \infty$, for each $x^* \in X^*$. Prove $\sup_{a \in A} \|a\| < \infty$.
6. Let $\mu$ and $\nu$ be finite measures such that each is absolutely continuous with respect to the other. Prove $\frac{d\mu}{d\nu} = \frac{1}{(d\nu/d\mu)}$ a.e. $\mu$.
7. Let $X, Y, Z$ be Banach spaces and let $\mathcal{F}$ be a total family of continuous linear maps on $X$ to $Y$. Let $T : Z \to X$ be a linear map such that $fT$ is continuous for every $f \in \mathcal{F}$. Prove $T$ is continuous. [Hint: Use closed graph theorem]

Note: $\mathcal{F}$ total means if $f(x) = 0$ for all $f \in \mathcal{F}$, then $x = 0$. 