
Print your name on each sheet turned in. Write all proofs in a neat and logical fashion.

1) State and prove the Radon-Nikodym Theorem.

2) State and prove the Vitali Integral Convergence Theorem.

3) Let \((X, \Sigma, \mu)\) be a finite measure space and suppose 
\(\Sigma = \sigma(\Sigma_0)\), where \(\Sigma_0\) is an algebra. Show that for 
every \(\epsilon > 0\), if \(E \in \Sigma\), then there exists an \(E_0 \in \Sigma_0\) 
such that \(\mu(E \Delta E_0) < \epsilon\)

4) Let \(Y\) and \(Z\) be closed subspaces in the B-space \(X\). 
Suppose each \(x \in X\) has a unique representation in 
the form \(x = y + z\), with \(y \in Y\) and \(z \in Z\). Show that 
there exists a constant \(K\) such that \(1y1 \leq K1x1\) and 
\(1z1 \leq K1x1\), for each \(x \in X\). [Hint: First apply Closed Graph Thm 
to \(T: X \to Y, T(x) = y\)].

5) Let \((X, \Sigma, \mu)\) be a measure space. Let \((f_n)\) be a sequence 
of integrable functions such that \(\int \Sigma f_n d\mu < \infty\).
Does \(\Sigma f_n\) converge a.e.? Prove.

6) Let \((\Omega, \mathcal{F}, P)\) be a probability space. Let \(X\) and \(Y\) 
be integrable random variables. Suppose \(X\) and \(Y\) are independent. 
Find \(E(X|Y)\).

7) Let \((X, \Sigma, \mu)\) be a finite measure space; let \((f_n)\) be a sequence 
of integrable functions such that \(\int f_n d\mu \to \) for each \(E \in \Sigma\) . 
Assume \((f_n)\) is \(L^1\) bounded. Show there exists an integrable function \(f\) such that \(\int f_n d\mu \to \int f d\mu \) 
for all \(E \in \Sigma\).