Be sure to write all proofs in a neat and logical fashion. Give reasons for all steps!

1. State and prove the Vitali Convergence Theorem for integrals.
2. State and prove the Martingale Convergence theorem.
3. Let \((X, \mathcal{B}, \mu)\) and \((Y, \mathcal{J}, \nu)\) be finite measure spaces. Construct \(\mu \otimes \nu\) on \(\mathcal{B} \otimes \mathcal{J}\). Give details.
4. Let \(\{\mu_n\}\) be a sequence of real valued signed measures on \((S, \Sigma)\). Define \(\lambda(A) = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{\mu_n(A)}{1 + \mu_n(S)},\) \(A \in \Sigma\). Show \(\lambda\) is a finite measure on \(\Sigma\) and \(\mu_n \leq \lambda\) for each \(n\).
5. Let \(f : [0, b] \to \mathbb{R}\) be Lebesgue integrable. Suppose \(\int_{[0,c]} f \, dm = 0\) for each \(c \in [0,b]\). Show \(f = 0\) a.e. \(m\).
6. Let \((S, F, P)\) be a pr. space, \(Y\) an integrable r.v. and \(\mathcal{G}\) a sub \(\sigma\)-field of \(F\). Suppose \(\mathcal{E}\) and \(F(Y)\) are independent. What is \(E(Y | \mathcal{E})\) ? Prove.
7. Let \(X\) be a B-space and suppose \(F\) and \(Z\) are closed linear subspaces of \(X\) such that each \(x \in X\) has a unique representation of the form \(x = y + z\), where \(y \in F\) and \(z \in Z\). Show that there is a constant \(K\) such that \(|y| \leq K|x|\) and \(|z| \leq K|x|\), for each \(x \in X\) where \(y \in F\) and \(z \in Z\) are given as above.

Hint: define a map \(T : X \to F\) which is useful, use the closed graph theorem to get continuity; then define \(S : X \to Z\), etc.