(i) State the following theorems:
1) The Hahn-Banach Theorem
2) The Egorov Theorem
3) The Vitali Convergence Theorem
4) The Radon-Nikodym Theorem
5) The Fubini Theorem
6) The Monotone Convergence Theorem
7) The Completeness Theorem for $L^1_F(\mu)$
8) The equality $|g|_1 = |g|_{1\mu}$.

(ii) Prove one of the theorems 3 or 7.

(iii) Solve the following problems, where $(X, \mathcal{F}, \mu)$ is a measure space and $F$ is a Banach space.

1) If $f_n \to f$ in $L^1_F(\mu)$ and $f_n \to g$, $\mu$-a.e., then $f = g$, $\mu$-a.e.
2) Let $\mathcal{F} \subseteq \mathcal{F}$ be a $\sigma$-algebra. Prove the existence of the conditional expectation $E(f|\mathcal{F})$ for $f \in L^1_F(\mu)$.
3) Let $\mathcal{G}$ be a $\sigma$-ring generating $\mathcal{F}$ and $f \in L^1_F(\mu)$. Prove that if $\int_A f \, d\mu = 0$ for every $A \in \mathcal{G}$, then $f = 0$, $\mu$-a.e.
4) Let $(X, \mathcal{F})$ and $(Y, \mathcal{T})$ be measurable spaces, $\mu : \mathcal{F} \to \mathbb{R}$ and $\nu : \mathcal{T} \to \mathbb{R}$, $\sigma$-additive measures. Prove the existence of the product measure $\mu \times \nu : \mathcal{F} \times \mathcal{T} \to \mathbb{R}$ satisfying $\mu \times \nu(A \times B) = \mu(A)\nu(B)$ for $A \times B \in \mathcal{F} \times \mathcal{T}$ and $\mu \times \nu(I) = \mu(I)\nu(I)$.
5) If $f \in L^1_F(\mu)$, there is a sequence $(f_n)$ of $\mathcal{F}$-step functions.
Note. Write complete computations and explanations.
Do not use arrows or other unnecessary signs. Use words for explanations.
Write complete sentences.