Present all work in a neat and logical fashion. Put each problem on a separate page. Print name on each sheet.

1. State and prove the Lebesgue Monotone Convergence Theorem.

2. Prove that $L^1(S, \Sigma, \mu)$ is complete, where $\mu$ is a nonnegative measure.

3. Give the construction of the product measure $\mu \times \nu$ on $S \times W$, where $(X, S, \mu)$ and $(Y, W, \nu)$ are $\sigma$-finite nonnegative measure spaces.

4. Let $f$ be Lebesgue integrable on $\mathbb{R}$. Suppose $\int_0^x f \, dm = 0$ for all real $x$. What can you say about $f$? Prove your answer.

5. Let $f$ be a Lebesgue integrable function on $\mathbb{R}$. Let $\epsilon > 0$. Is it possible to find an interval $I$ such that whenever $E$ is a measurable set such that $E \cap I = \emptyset$, then $|\int_E f \, dm| < \epsilon$?

6. Let $X$ be a metric space. Let $\{P_n\}$ be a sequence of regular probability measures on $B(X)$, the Borel subsets of $X$. Let $P = \sum_{n=1}^{\infty} \left( \frac{1}{2^n} \right) P_n$. Show that $P$ is regular on $B(X)$.

7. Let $\mu(dx) = x^2 \, dx$ on $[0, 1]$. Let $T : [0, 1] \to [0, 1]$ be defined by $T(x) = x^4$. Compute the Radon-Nikodym derivative of the image measure $T\mu$.

8. Let $(\mu, F, \mu)$ be a measure space with $\mu(\Omega) = 1$.

Let $A$ and $B$ be sub $\sigma$-algebras of $F$:

Suppose $\mu(A \cap B) = \mu(A)\mu(B)$ for all $A \in A$ and $B \in B$. Prove that $\int f g \, d\mu = (\int f \, d\mu)(\int g \, d\mu)$ for all $A$-measurable positive functions $f$ and all $B$-measurable positive functions $g$. 