Logic Qualifying Exam, May 2017

Answer six questions, at least one in each section.

Section 1
1. State and prove Tarski’s criterion for elementarity of submodels.
2. State and prove the downward Loewenheim–Skolem theorem.
3. Prove that the models \( \langle \mathbb{Z}, \leq \rangle \) (integers with the usual ordering) and \( \langle \mathbb{Z} + \mathbb{Z}, \leq \rangle \) (two copies of integers with the usual ordering, one following the other) are elementarily equivalent.

Section 2
1. Prove that for any cardinal \( \kappa \), the cofinality \( cf(2^\kappa) > \kappa \).
2. Show that the wellordering principle implies the Zorn’s lemma.
3. Show that a c.c.c. forcing preserves all cardinals.

Section 3
1. Show that every computably enumerable set has an infinite computable subset.
2. Show that there are two subsets of \( \omega \) which are incomparable in the sense of the Turing ordering.
3. Explain what a \( \Pi^1_1 \) complete set is and provide an example with a proof.