1. Let \((f_n)_{n=0}^\infty\) be a sequence of differentiable functions. Decide whether each implication is valid, giving proof or counterexample as appropriate:
   (i) if \(f_n \rightarrow f\) uniformly on \((-\infty, \infty)\) then \(f_n^2 \rightarrow f^2\) uniformly on \((-\infty, \infty)\);
   (ii) if \(f_n \rightarrow f\) uniformly on \([-1, 1]\) then \(f_1^1 f_n \rightarrow f_1^1 f\);
   (iii) if \(f_n \rightarrow f\) uniformly on \((-1,1)\) then \(f\) is differentiable and \(f'_n \rightarrow f'\) uniformly on \((-1,1)\).

2. Let \(f\) be a continuous real-valued function on \([0,1]\) and let \(\alpha \in (0,\infty)\).
   Assume that
   \[
   \int_0^1 t^{\alpha} f(t) \mathrm{d}t = 0
   \]
   for all but finitely many values of \(n \in \mathbb{N}\). What conclusions can be drawn about \(f\)?

3. Let \(\mathcal{F}\) be an equicontinuous family of real-valued functions on the compact metric space \(X\). Denote by \(A \subseteq X\) the set whose elements are precisely those \(a \in X\) at which \(\mathcal{F}\) is bounded in the sense that \(\{f(a) : f \in \mathcal{F}\} \subseteq \mathbb{R}\) is bounded. Prove that \(A\) is both open and closed.

4. Let \((f_n)_{n=0}^\infty\) be a sequence of measurable real-valued functions. Decide whether each of the following sets is measurable:
   (i) \(\\{\omega : (f_n(\omega))_{n=0}^\infty\text{ is unbounded}\}\);
   (ii) \(\\{\omega : (f_n(\omega))_{n=0}^\infty\text{ is periodic}\}\);
   (iii) \(\\{\omega : (f_n(\omega))_{n=0}^\infty\text{ has distinct terms}\}\).

5. Let \((\Omega, \mathcal{A}, \mu)\) be a measure space that is finite in the sense that \(\mu(\Omega) < \infty\).
   Let \((f_n)_{n=0}^\infty\) be a sequence of non-negative measurable functions converging pointwise to \(f\) on \(\Omega\). True or false (proof or counterexample):
   \[
   \int_\Omega \frac{1}{1 + f_n} \, d\mu \rightarrow \int_\Omega \frac{1}{1 + f} \, d\mu \text{ as } n \rightarrow \infty.
   \]