1. Let $F$ be a closed subset of the reals $\mathbb{R}$ with the usual topology. Show $F$ is a countable union of compact sets.

2. Let $E$ be a nonempty subset of a metric space. State the definition of the distance $\rho_E(x)$ of a point $x$ to $E$. Characterize $\overline{E}$ in terms of $\rho_E$, where $\overline{E}$ is the closure of $E$.

3. (a) Let $f$ be a continuous function from a compact metric space $X$ into a metric space $Y$. Prove $f(X)$ is compact.
   (b) Assume the setting in (a) with the further assumption that $Y$ is the reals with the usual topology. Show that $f$ assumes maximum and minimum values on $X$.

4. Let $f$ be a continuous map of a metric space $X$ into a metric space $Y$.
   (a) Show that if $f$ is NOT uniformly continuous on $X$, then for some $\varepsilon > 0$ there are sequences $\{p_n\}, \{q_n\}$ in $X$ such that $d_Y(f(p_n), f(q_n)) > \varepsilon$ for each $n$ but $d_X(p_n, q_n) \to 0$.
   (b) Assume that $X$ is compact. Show, using (a), that $f$ is uniformly continuous on $X$.

5. Let $\{x_n\}$ be a sequence of points in $(a,b)$ and let $\{c_n\}$ be a sequence of positive numbers such that $\sum c_n$ converges. Define
   \[
   f(x) = \sum_{n:x_n<x} c_n, \quad a < x < b,
   \]
   where the summation is understood as follows: sum over those indices $n$ for which $x_n < x$. If there are no points $x_n < x$, define the sum to be zero.
   (a) Show $f(x^-) = f(x)$ for each $x$ in $(a,b)$.
   (b) Show $f(x_+^n) - f(x_-^n) = c_n$, for each $n$.

6. Suppose $f$ is continuous on $[0, \infty)$, differentiable on $(0, \infty)$, $f(0) = 0$ and $f'$ is monotonically increasing. Define $g$ on $(0, \infty)$ by $g(x) = f(x)/x$, $x > 0$. Prove $g$ is monotonically increasing.