1. Give an example of each of the following, with some explanation. 
   (a) (3 points) A non-solvable finite group.
   (b) (3 points) A non-cyclic infinite abelian group.
   (c) (4 points) A non-abelian solvable finite group.

2. (10 points) Let $G$ be a finite group and suppose $G$ acts transitively on a set $\Omega$. Prove that $|\Omega|$ (the number of elements in $\Omega$) is finite and that $|\Omega|$ divides $|G|$. 

3. (10 points) Let $G$ be a group, and let $A$ and $B$ be normal subgroups of $G$ such that $A \cap B = 1$. Prove that every element of $A$ commutes with every element of $B$. 

4. (10 points) Let $G$ be a group of order 14 and let $H$ be a (multiplicative) cyclic group of order 4. Suppose that $\phi : G \to H$ is a group homomorphism. Let $g \in G$. Prove that there exists some $h \in H$ such that $\phi(g) = h^2$. 

5. (10 points) Let $a = (1\ 2\ 3\ 4)(5\ 6)(7\ 8)$ and let $b = (7\ 2\ 8\ 4)(1\ 6)(3\ 5)$ be elements of the symmetric group $S_8$. Is there an element $\sigma \in S_8$ such that $\sigma a \sigma^{-1} = b$? If so find one. 

6. (10 points) Prove that if $G$ is a group of order 56, then $G$ has a normal Sylow 2-subgroup or a normal Sylow 7-subgroup.