1. Give an example of each of the following, with some explanation.
   (a) (3 points) A solvable group of order greater than 10.
   (b) (3 points) A non-cyclic abelian group.
   (c) (4 points) A finite group having a non-cyclic abelian proper subgroup.

2. (10 points) Let $G$ be a finite group. Suppose that $N$ is a normal subgroup of even order of $G$ such that the non-trivial elements of $N$ form a single conjugacy class of $G$. Prove that $N$ is abelian.

3. (10 points) Let $G = A_5$ be the alternating group on 5 letters. Prove that $G$ is a simple group.

4. (10 points) Let $G = D_{2n}$ be the dihedral group of order $2n$. Suppose that $n \geq 4$ is a power of 2. Prove that $G$ is nilpotent and describe the nilpotency class of $G$.

5. (10 points) Let $G$ be a group of order 30. Prove that $G$ has a normal subgroup of order 15.

6. (10 points) State and prove Sylow’s First Theorem.