1. (10 points) State and prove Sylow’s First Theorem.

2. Consider the symmetric group $S_6$, and the corresponding alternating group $A_6$.
   (a) (5 points) How many elements of order 4 are there in $S_6$? Justify your answer.
   (b) (5 points) How many elements of order 4 are there in $A_6$? Justify your answer.

3. Let $G$ be the alternating group $A_{100}$, and assume $G$ acts on a finite set $\Omega$.
   (a) (5 points) Prove that, if $|\Omega| = 90$, then $G$ acts trivially on $\Omega$, i.e. $G$ has 90 orbits of size 1 on $\Omega$.
   (b) (5 points) Prove that if $|\Omega| = 120$ and $G$ does not act trivially on $\Omega$, then $G$ has, at most, 21 orbits on $\Omega$.

4. (10 points) Prove that every group of order 30 has a normal subgroup of order 15.

5. (10 points) Let $G$ be a group, let $N$ be a normal subgroup of $G$, and let $H$ be a subgroup of $G$. Assume that $H$ is solvable. Prove that $HN/N$ is a solvable group.

6. (10 points) Let $G$ be a group of order 39. Does $G$ need to be abelian? Prove that your answer is correct.